

Matrix A	Matrix B	Answer
1 3 4		-11/49 4/49 5/49
5 7 6		-2/7 2/7 -1/7
8 1 4		51/98 -23/98 4/49

Solution

The inverse of a matrix, or the reciprocal matrix, can be found in three steps. The first step required is to find the determinant of M, or $|M|$. The second step involves finding the adjugate of M, or $\text{adj}(M)$. The third and final step is to scalar divide $\text{adj}(M)$ by $|M|$.

Step 1, calculate $|M|$:

To find $|M|$, which is greater than $[2 \times 2]$, the minors of the first row of matrix M must be found by applying *determinant expansion by minors*, which will reveal the following minor matrices:

M.1	and	M.2	and	M.3
1 3 4		1 3 4		1 3 4
5 7 6		5 7 6		5 7 6
8 1 4		8 1 4		8 1 4

After re-writing the minor matrices we can calculate their determinants:

M.1	and	M.2	and	M.3
7 6		5 6		5 7
1 4		8 4		8 1

Find the determinant of the minor matrix M.1:

For a $[2 \times 2]$ matrix, the following formula is used to calculate the determinate:

$$\text{If matrix } M = \begin{matrix} a & b \\ c & d \end{matrix}$$

$$\text{Then } |M| = (a * d) - (b * c)$$

Therefore, to calculate the determinate for M.1, multiply a by b then subtract the product of b multiplied by c, which will result in:

$$(7 * 4) - (6 * 1)$$

$$|M.1|: 22$$

Find the determinant of the minor matrix M.2:
 $(5 * 4) - (6 * 8)$

$$|\mathbf{M.2}|: -28$$

Find the determinant of the minor matrix M.3:
 $(5 * 1) - (7 * 8)$

$$|\mathbf{M.3}|: -51$$

Get cofactors by applying the following signing pattern to the matrix:

$$\begin{array}{ccc} + & - & + \\ - & + & - \\ + & - & + \end{array} \quad \begin{array}{l} \text{applied} \\ \text{to} \end{array} \quad \begin{array}{ccc} 1 & 3 & 4 \\ 5 & 7 & 6 \\ 8 & 1 & 4 \end{array} = \begin{array}{l} 1; \\ -3; \\ 4 \end{array}$$

Calculate the determinant by multiplying each cofactor by a corresponding minor matrix determinant:

$$(1 * 22) + (-3 * -28) + (4 * -51)$$

$$|\mathbf{M}|: -98$$

Step 2, calculate adj(M):

To find adj(M), which is greater than [2 x 2], the minors of the matrix M must be found by applying *determinant expansion by minors*, which will reveal the following minor matrices:

M.1	M.2	M.3
1 3 4	1 3 4	1 3 4
5 7 6	5 7 6	5 7 6
8 1 4	8 1 4	8 1 4

M.4	M.5	M.6
1 3 4	1 3 4	1 3 4
5 7 6	5 7 6	5 7 6
8 1 4	8 1 4	8 1 4

M.7	M.8	M.9
1 3 4	1 3 4	1 3 4
5 7 6	5 7 6	5 7 6
8 1 4	8 1 4	8 1 4

After finding the minor matrices we calculate their determinants:

Find the determinant of the minor matrix M.1:
 $(7 * 4) - (6 * 1)$

$$|\mathbf{M.1}|: 22$$

Find the determinant of the minor matrix M.2:
 $(5 * 4) - (6 * 8)$

$$|\mathbf{M.2}|: -28$$

Find the determinant of the minor matrix M.3:
 $(5 * 1) - (7 * 8)$

$$|\mathbf{M.3}|: -51$$

Find the determinant of the minor matrix M.4:
 $(3 * 4) - (4 * 1)$

$$|\mathbf{M.4}|: 8$$

Find the determinant of the minor matrix M.5:
 $(1 * 4) - (4 * 8)$

$$|\mathbf{M.5}|: -28$$

Find the determinant of the minor matrix M.6:
 $(1 * 1) - (3 * 8)$

$$|\mathbf{M.6}|: -23$$

Find the determinant of the minor matrix M.7:
 $(3 * 6) - (4 * 7)$

$$|\mathbf{M.7}|: -10$$

Find the determinant of the minor matrix M.8:
 $(1 * 6) - (4 * 5)$

$$|\mathbf{M.8}|: -14$$

Find the determinant of the minor matrix M.9:
 $(1 * 7) - (3 * 5)$

$$|\mathbf{M.9}|: -8$$

From the determinants of minors create a new minor matrix:

$$\begin{array}{|c|c|c|} \hline \mathbf{M.1} & \mathbf{M.2} & \mathbf{M.3} \\ \hline \mathbf{M.4} & \mathbf{M.5} & \mathbf{M.6} \\ \hline \mathbf{M.7} & \mathbf{M.8} & \mathbf{M.9} \\ \hline \end{array} = \begin{array}{ccc} 22 & -28 & -51 \\ 8 & -28 & -23 \\ -10 & -14 & -8 \end{array}$$

Get the cofactors matrix by applying the following signing pattern to the minor matrix:

$$\begin{array}{ccc} +1 & -1 & +1 \\ -1 & +1 & -1 \\ +1 & -1 & +1 \end{array} * \begin{array}{ccc} 22 & -28 & -51 \\ 8 & -28 & -23 \\ -10 & -14 & -8 \end{array} = \begin{array}{ccc} 22 & 28 & -51 \\ -8 & -28 & 23 \\ -10 & 14 & -8 \end{array}$$

Create the adjugate by taking each row of the cofactors matrix and transforming it to a column in the adjugate matrix:

$$\begin{array}{ccc} 22 & 28 & -51 \\ -8 & -28 & 23 \\ -10 & 14 & -8 \end{array} \text{ becomes } \begin{array}{ccc} 22 & -8 & -10 \\ 28 & -28 & 14 \\ -51 & 23 & -8 \end{array}$$

adj(M):

$$\begin{array}{ccc} 22 & -8 & -10 \\ 28 & -28 & 14 \\ -51 & 23 & -8 \end{array}$$

Step 3, scalar divide adj(M) by |M|:

$$\begin{array}{ccc} 22 & -8 & -10 \\ 28 & -28 & 14 \\ -51 & 23 & -8 \end{array} / -98 =$$

Divide each element in adj(M) by -98:

$$\begin{array}{ccc} (22 / -98) & (-8 / -98) & (-10 / -98) \\ (28 / -98) & (-28 / -98) & (14 / -98) \\ (-51 / -98) & (23 / -98) & (-8 / -98) \end{array}$$

Division result:

$$\begin{array}{ccc} -11/49 & 4/49 & 5/49 \\ -2/7 & 2/7 & -1/7 \\ 51/98 & -23/98 & 4/49 \end{array}$$

Answer:

-11/49 4/49 5/49
-2/7 2/7 -1/7
51/98 -23/98 4/49